Winning Losers in the Italian Football League “Auction” for Co-Ownership Resolution

Nicola Dimitri

July 2012
The Maastricht School of Management is a leading provider of management education with worldwide presence. Our mission is to enhance the management capacity of professionals and organizations in and for emerging economies and developing countries with the objective to substantially contribute to the development of these societies.

www.msm.nl

The views expressed in this publication are those of the author(s) and do not reflect the official policy or position of the government of India. Publication does not imply endorsement by the School or its sponsors, of any of the views expressed.
WINNING LOSERS IN THE “ITALIAN FOOTBALL LEAGUE AUCTION” FOR CO-OWNERSHIP RESOLUTION

Abstract

In the Italian Football League rights to a player’s performance could be co-owned by two clubs for one year. Co-ownership must then be resolved and if the clubs fail finding an agreement they are asked by the League to participate to an auction, where each of them submits a price offer for the missing half of the rights. The player offering the highest price obtains the missing half of the rights paying that price to the opponent. In the paper we characterize the auction equilibrium structure with both complete and incomplete information. Due to its features, a main finding in such auction is that “losers” can obtain a higher payoff than “winners”, and in this sense be the real winners. Then, by considering the auction as a more general mechanism for co-ownership resolution, we extend the model to any finite number of players and argue how some of the results with two players do not necessarily generalize. In particular, while with two players the equilibrium expected payoffs can never be negative this may not be so with a higher number of players. Finally, also with incomplete information the symmetric bidding equilibrium function with any finite number of players is in the “winning losers” spirit. Indeed, bids range between one’s value and twice of it, and increase with the number of players since it becomes more likely that some opponent will have a higher value.

1. Introduction

In the Italian Football League property rights over a player’s performance could be co-owned by two Football clubs, say A and B. Upon having decided to share the ownership A and B agree on how to allocate the player’s performance for the current football season. The player rights could be temporarily transferred to a third club C, for free or for an agreed upon rate which the clubs share, or else the player may perform for A (B) under a specific agreement.

Clubs may opt for such form of co-ownership to undertake strategic investments over interesting players, at reduced costs. Yet, shared ownership can only last for one season since, after that, the Italian Football League imposes the two clubs to resolve it. Resolution takes place as follows. The two clubs first negotiate trying to find a consensus on how to resolve the issue. Successful negotiation would then allocate the full property to one of the two clubs. However, should they fail reaching an agreement the Italian Football League asks the two clubs to resolve the co-ownership according to the following simple, sealed bid, auction mechanism. Club A will make a price offer for the 50% rights owned by B and
so would B for the 50% owned by A. The club proposing the highest offer wins the auction, obtaining the 100% rights over the player performance, and pays the price offered to the other club to acquire the missing half of the rights.

Though a variation of a first price mechanism, to our knowledge the strategic connotations of such auction for co-ownership resolution have not yet been investigated. This may be due to its rarity and specificity, also its main point of interest, differentiating it from standard auctions, since participants enter the competition without a pre-specified role, as they could potentially end up buying or selling. Therefore unlike standard mechanisms, where roles are defined before competition takes place, the outcome of the auction in determining the winner will also define him as the buyer while the loser will be the seller.

Such specificity has a number of interesting strategic consequences. In the private values and complete information framework that we consider losers can end up enjoying a higher benefit than winners, in this sense being the “real” winners. The intuition is simple: with complete information in a pure strategy equilibrium players submit prices between their values, as if they would bargain for the item. In our model this may lead to an infinity of equilibria, with players offering the same price within their two values, so that when the winning bid is closer to the highest evaluation of the item the loser’s profit is larger than the winner’s. The pure strategy equilibrium structure in this case enjoys a continuity property, since when players’ values coincide then there exists a unique pure strategy Nash Equilibrium.

Interestingly, mixed strategy Nash Equilibria are defined only below the lowest value of the two values, namely outside the price interval supporting pure strategy equilibria. Intuitively, when the player with the lowest evaluation is uncertain as to which price the opponent bids, then he would find it convenient to limit the risk of winning with a too high a bid, and submit an offer below his own value. This way, in equilibrium, both players will enjoy positive payoffs, however with the player having the higher value always receiving a higher payoff.

Therefore, whether with pure or mixed strategy equilibrium, the two players never obtain negative profits.

By seeing this as a general mechanism to solve co-ownership, in the paper we also extend the IFLA to any finite number of players, and discuss how in such extension some of the findings valid with two players do not carry through. In particular, now there may be equilibria where players make negative profits, because the winning price has to be distributed among all the opponents. The structure of the paper is as follows. In Chapter 2 we provide an extensive strategic characterization of the complete information model with at least two players. In Chapter 3 we characterize the symmetric equilibrium bidding function with incomplete information, while Section 4 concludes the work.
2. The Model with Private Values and Complete Information

Consider an infinitely divisible object and let \( N \geq 2 \) be the number, as well as the set, of players. Let \( V_i \) be player \( i \in N \) evaluation of the object which is co-owned by the players with \( 0 \leq s_i \leq 1 \), such that, \( \sum_{i=1}^{N} s_i = 1 \), being the share of the object owned by agent \( i \). Players resolve such co-ownership in an auction, bidding for the missing part of the object with value \( v_i = (1 - s_i)V_i \). With no major loss of generality we assume \( v_1 \geq v_2 \geq \ldots \geq v_N \). The player submitting the highest price wins the auction and pays that price to the other players. However, while with two players the loser receives the entire winning price, with at least three players the winning price will be distributed among all the opponents, according to the scheme specified in the winning bid.

In the section we investigate the main strategic features of the game with complete information. To do so we first gain some, initial, insights of the model starting with the case of the Italian Football League, \( N = 2 \) and \( s_1 = \frac{1}{2} \), which we then extend to any finite \( N > 2 \) and \( 0 \leq s_i \leq 1 \). However, as we shall see going from two to a higher number of players will imply some meaningful changes in the equilibrium structure.

Inspired by the tie-break rule adopted by the Italian Football League, in what follows we assume that in case of ties player 1 keeps the object, otherwise player 2 keeps it and so on. A discussion of the equilibrium structure of the model when in case of ties the object goes to 2, as well as when the tie breaking rule is randomized, will be postponed until Section 2.3.

2.1 The Italian Football League Auction (IFLA): \( N = 2 \) and \( s_i = \frac{1}{2} \)

In this simplest setting players value \( v_i = (1 - s_i)V_i = (V_i/2) \), with \( i = 1,2 \), the missing part of the object for which each of them bids in the auction. If \( b_i \) is player \( i \)'s price bid for the missing half, then the player offering the highest price obtains the other half of the object, paying to the opponent the winning price. Hence, except for Section 2.4, we shall assume the cost of the already owned half of the object as sunk, outside the scope of the analysis.

Therefore, the profit function of player 1 is defined by

\[
\Pi_1(b_1) = \begin{cases} 
(v_1 - b_1) & \text{if } b_1 \geq b_2 \\
(b_2 - v_1) & \text{if } b_1 < b_2 
\end{cases}
\]

while for player 2 is
\[ \Pi_2(b_2) = \begin{cases} (v_2 - b_2) & \text{if } b_2 > b_1 \\ (b_1 - v_2) & \text{if } b_2 \leq b_1 \end{cases} \]

Since the auction is not all-pay, a main feature of IFLA reflected in the above payoff functions is that, for player \( i \), when the opponent bids above \( v_i \) he would be better off by losing the game. This point is embodied in the following proposition.

**Proposition 1** Consider an IFLA. If \( v_1 > v_2 \) then there is a continuum of Pure Strategy Nash Equilibria (PSNE) of the kind \( (b_1 = b; b_2 = b) \) with \( v_2 \leq b \leq v_1 \) where player 1 wins the object. If \( v_1 = v = v_2 \) there is a unique PSNE in which \( b_1 = v = b_2 \) and player 1 obtains the object.

**Proof** Start with \( v_1 = v = v_2 \) and consider, say, \( b_2 > v \). Then, because of the tie break rule player 1’s best reply would be \( b_1 = b_2 - \epsilon \), with \( \epsilon > 0 \) small enough. But then player 2 will in turn slightly undercut player 1 and so on until \( b_1 = v = b_2 \). A symmetric reasoning would hold starting from \( b_2 < v \), where now both players outbid, rather than undercut, each other, until \( b_1 = v = b_2 \), proving this to be the unique PSNE.

Suppose now \( v_1 > v_2 \). Then, at any \( b_2 > v_1 \) for player 1 would be best bidding \( b_1 = b_2 - \epsilon \), with \( \epsilon > 0 \) small enough. The same would be optimal for player 2, as in the previous point. A symmetric argument applies considering \( b_2 < v_2 \). Hence, it is easy to check that each element of the pair \( (b_1 = b, b_2 = b) \), with \( v_2 \leq b \leq v_1 \) is best reply against the other and so each such pair is a PSNE.

The above proposition suggests that when \( v_1 > v_2 \), in any pure strategy equilibrium the IFLA is efficient as the object goes to the player with the highest evaluation. Moreover players behave as bargainers, that is bids lie between their two values. For this reasons, the model may give rise to apparently unusual outcomes, for an auction, in particular that the “loser” may obtain a higher payoff than the “winner”, the former being the seller while the latter the buyer, a point formalized by the following corollary.

**Corollary 1** If \( v_1 > v_2 \) then \( \Pi_2(b) \geq \Pi_1(b) \) for PSNE \( (b_1 = b; b_2 = b) \) and with \( \frac{v_1 + v_2}{2} \leq b \leq v_1 \) and \( \Pi_1(b) > \Pi_2(b) \) if \( v_2 \leq b < \frac{v_1 + v_2}{2} \).

### 2.2 The Model with \( N > 2 \) players and shares \( 0 \leq s_i \leq 1 \)

We now extend the IFLA to a model with \( N > 2 \) players and shares \( 0 \leq s_i \leq 1 \). As above, if \( V_i \), with \( i = 1, \ldots, N \), is the object value for player \( i \) then \( v_i \) now is defined by

\[ v_i = (1 - s_i)V_i \]

Moreover, player \( i \) now expresses his bid as an \( N - 1 \) dimensional vector
\[ b_i = (b_{i1}, \ldots, b_{ii-1}, b_{ii+1}, \ldots, b_{iN}) \]

(with a slight abuse of notation) such that

\[ b_i = \sum_{j \neq i} b_{ij} \quad (4) \]

where \( b_{ij} \) is the offer made by player \( i \) to player \( j \).

Defining \( b_{(1)} \) as the winning bid, then \( b_{(1)} = (b_{(1)1}, \ldots, b_{(1)N}) \) is the vector of offers made by the winner to the \( N - 1 \) opponents, with the component \( b_{(1)j} \) representing the offer made to player \( j \). Hence, player \( i \)'s payoff function now becomes

\[ \Pi_i(b_i) = \begin{cases} (v_i - b_i) & \text{if } b_i = b_{(1)} \\ (b_{(1)i} - v_i) & \text{otherwise} \end{cases} \]

In what follows we shall see how the model exhibits a number of different strategic features with respect to IFLA, the first two of which are contained in the next pair of results.

**Proposition 2** Assume \( N > 2 \) and \( v_1 = \cdots = v_k = v > v_{k+1} \geq \cdots \geq v_N \). (a) If \( k > 2 \) then at all PSNE the winner obtains negative profits while (b) if \( k \geq 2 \) then at no PSNE the winner obtains positive profits.

**Proof** (a) We prove the result by contradiction, showing that there cannot exist PSNE with \( b_{(1)} \leq v \).

Suppose first there exists a PSNE where \( b_{(1)} = v \) and that \( b_{(1)j} = v \) for some \( j \).

If \( v_j \leq v \) then, for at least another player \( i \neq j \), it is \( v_i = v \) and \( b_{(1)i} = 0 \). Hence player \( i \) would obtain \( \Pi_i(b_{(1)i}) = b_{(1)i} - v = -v < 0 \). But then he would be better-off by bidding \( b_i = v + \epsilon \), with \( \epsilon > 0 \) small enough, to win the auction and get a payoff equal to \( \Pi_i(b_i = v + \epsilon) = v - v - \epsilon = -\epsilon > 0 \).

Similarly, if \( b_{(1)j} < v \) for all \( j \), then, again, for at least one player \( i \) it would be \( v_i = v \) and \( 0 \leq b_{(1)i} < v \). Hence, \( \Pi_i(b_{(1)i}) = b_{(1)i} - v < 0 \) and player \( i \) would be better off by bidding \( b_i = v + \epsilon \), with \( \epsilon > 0 \) small enough, to win the auction and obtain payoff \( \Pi_i(b_i = v + \epsilon) = v - v - \epsilon = -\epsilon > b_{(1)i} - v \).

Suppose now there exists a PSNE where \( b_{(1)} < v \). Then for at least one player \( i \) it is \( v_i = v, 0 \leq b_{(1)i} < v \) and \( \Pi_i(b_{(1)i}) = b_{(1)i} - v < 0 \). But for such player it would be better to bid \( b_i = b_{(1)} + \epsilon \), with \( \epsilon > 0 \) small enough, to win the auction and obtain a positive payoff equal to \( \Pi_i(b_i = b_{(1)} + \epsilon) = v - b_{(1)} - \epsilon > 0 \).

(b) We now show that with \( k = 2 \) it is \( b_{(1)} \geq v \). By the same argument as in point (a) it cannot be \( b_{(1)} < v \). Then, the following example illustrates that there could be PSNE where
the winner obtains zero profits. Indeed suppose \( v_1 = v_2 = v > v_3 = \ldots = v_N = \varepsilon \), with \( \varepsilon > 0 \) small enough. Then the profile of bids \( b_1 = (b_{12} = v, b_{13} = 0, \ldots, b_{1N} = 0), \ b_2 = (b_{21} = v, b_{23} = 0, \ldots, b_{2N} = 0) \) and \( b_i = (b_{i1} = v, b_{i2} = 0, \ldots, b_{iN} = 0) \), for \( i > 2 \) is a PSNE where player 1 wins the auction obtaining zero profit.

If the above proposition identifies a first meaningful difference with IFLA, that the winner typically loses in case of equal values, The following corollary further articulate such difference.

**Corollary 2** Suppose \( N > 2 \) and assume \( v_1 = \ldots = v_N = v \), with \( k > 2 \). Then no profile of bids of the type \( b_1 = \ldots = b_N = v \) is a PSNE.

Based on Proposition 3 as well as on Corollary 3, it is then natural to ask if when \( v_1 = \ldots = v_k = v \geq v_{k+1} \geq \ldots \geq v_N \) PSNE could at all exist. The answer is yes and the following result exemplifies a particular equilibrium where all players obtain negative payoffs.

**Proposition 4** Suppose and \( v_1 = \ldots = v_N = v \). Then the profile of bids

\[
b_1 = (b_{12} = \frac{b}{N-1}, \ldots, b_{1N} = \frac{b}{N-1}), b_2 = (b_{21} = 0, \ldots, b_{2[N-1]} = 0, b_{2N} = b), \ldots, b_N = (b_{N1} = 0, \ldots, b_{N[N-2]} = 0, b_{N[N-1]} = b) \text{ with } \frac{2(N-1)}{N} v < b < 2v, \text{ is a PSNE where all players obtain negative payoffs.}
\]

**Proof** Indeed, suppose \( b_1 = \left( b_{12} = \frac{b}{N-1}, \ldots, b_{1N} = \frac{b}{N-1} \right) \), with \( \frac{2(N-1)}{N} v < b < 2v \). Then losing the auction would be best for all players \( i > 1 \) if \( b_{1i} - v \geq v - b \). But since \( b_{1i} = \frac{b}{(N-1)} \), then it must be \( \frac{b}{(N-1)} - v \geq v - b \), which is satisfied for \( b \geq \frac{2(N-1)}{N} v \).

Moreover, since \( b < 2v \) then \( \frac{b}{(N-1)} - v < 0 \) implies \( b < (N-1)v \) which holds true.

Analogously, against \( b_i = (b_{i1} = 0, \ldots, b_{iN} = b) \) for player 1 is best to win the auction with \( b_1 = b \geq \frac{2(N-1)}{N} v > v \), since against the opponents’ bids \( \Pi_i(b_1 = b) = v - b < 0 \) and \( v - b > -v = \Pi_1(0) \) given that \( b < 2v \).

In what follows we discuss some main features of the PSNE when players have different values.

**Proposition 4** Suppose \( N > 2 \) and (a) assume \( v_1 > 2v_2 > v_2 \geq v_3 \geq \ldots \geq v_N \). Then at all PSNE player 1 wins the auction and it is \( \frac{2 \sum_{i=1}^N v_i}{N} \leq b_{(1)} = b_1 \leq \text{Min}_{i \neq 1} (2v_1 - b_{i1}) \), where \( b_{i1} \) is the offer made in equilibrium by player \( i \) to player 1. If (b) \( 2v_2 > v_1 > v_2 \geq v_3 \geq \ldots \geq v_N \) then there could be PSNE where player 1 does not win.
Proof (a) At any PSNE in which player \( i \) wins the auction, for any player \( j \neq i \) it must be true that \( b_{ij} - v_j \geq v_j - b_i \), namely \( j \) should not find it profitable to outbid \( i \) and win the auction. It follows that \( b_{ij} \geq 2v_j - b_i \) and, because of (3), \( 2v_j - b_i \leq b_i \) implying \( v_j \leq b_i \). Moreover, for \( i \) to win it must be \( v_i - b_i \geq Max_j b_{ji} - v_i \), hence \( b_i \leq Min_j (2v_i - b_{ji}) \). Therefore, since \( v_1 > 2v_2 \) except for player \( i = 1 \), no \( b_i \) can satisfy the condition \( v_j \leq b_i \leq Min_j (2v_i - b_{ji}) \leq 2v_i \), for all \( j \neq i \).

Finally, considering player 1 and summing up both sides of \( b_{1i} - v_i > v_i - b_1 \) with respect to \( i \) it follows that \( b_1 \geq 2 \sum_{i=1} v_i - (N - 1)b_1 \) and that \( b_1 \geq \frac{2 \sum_{i=1} v_i}{N} \)

(b) It is immediate to check that
\[
b_1 = (b_{12} = 0, \ldots, b_{1N} = 0), \quad b_i = (b_{i1} = v_1, b_{i2} = 0, \ldots, b_{iN} = 0) \quad \text{for} \quad i > 1
\]
is a PSNE in which player 2 wins the auction, with player 1 obtaining zero profit, player 2 gets profits \( v_2 - v_1 < 0 \) and the other players \(-v_j < 0\). Indeed, against the opponents’ profile of bids, for player 1 is best not to win as he would get negative, rather than zero, profits, while losing the auction for player \( i > 1 \) would mean to obtain \(-v_i \) which, since \( 2v_2 > v_1 \), it is lower than \( v_i - v_1 < 0 \), what player 2 obtains winning the auction.

2.3 The IFLA With A Randomized Tie-Breaking Rule

In this section we briefly discuss the IFLA with an alternative, randomized, tie-breaking rule, which in case of ties “draws” the winner with probability \( \frac{1}{2} \). As a consequence, the profit function of player 1 will now be defined by

\[
\Pi_1(b_1) = \begin{cases} 
\frac{(v_1 - b_1)}{(b_1 - b_2)} = 0 & \text{if } b_1 > b_2 \\
\frac{2}{(b_2 - v_1)} & \text{if } b_1 = b = b_2 \\
0 & \text{if } b_1 < b_2 
\end{cases}
\]

while for player 2 is

\[
\Pi_2(b_2) = \begin{cases} 
\frac{(v_2 - b_2)}{(b_2 - b_1)} = 0 & \text{if } b_2 > b_1 \\
\frac{2}{(b_1 - v_2)} & \text{if } b_1 = b = b_2 \\
0 & \text{if } b_2 < b_1 
\end{cases}
\]

It is immediate to check that when \( v_1 \neq v_2 \) now the only two PSNE pairs are \( (b_1 = v_1; b_2 = v_1) \) and \( (b_1 = v_2; b_2 = v_2) \) while when \( v_1 = v = v_2 \) the unique PSNE is still \( (b_1 = v; b_2 = v) \)
In this sense, a randomized tie-breaking rule is “acting as a device” to select two equilibria in the set of PSNE identified by Proposition 1, where one of the players will obtain the entire rent while the other zero profits.

3. The Model with Private Values and Incomplete Information

Suppose now players' values $v_i$, with $i = 1, \ldots, N$, are private information. In particular, assume them to be i.i.d random variables with marginal distribution function $G(v_i)$ and density $G'(v_i) = g(v_i)$. Then, adopting the notation and approach by Krishna-Morgan (1997) and Krishna (2009), in what follows we characterize the increasing, symmetric, equilibrium bidding function

$$\beta_i(v) = \beta(v) = (\beta_{i1}(v), \beta_{(i-1)1}(v), \beta_{(i+1)1}(v), \ldots, \beta_{iN}(v)),$$

where $\beta_{ij}(v)$ is the offer made by bidder $i$ to bidder $j$, with $i \neq j$. Since at a symmetric equilibrium it is also $\beta_{ij}(v) = \beta_{kj}(v)$, for all $i \neq j \neq k$, it follows that with $N$ players, each bidder’s equilibrium vector of bids must be given by

$$\beta(v) = \left(\frac{\beta(v)}{N-1}, \ldots, \frac{\beta(v)}{N-1}\right)$$

That is, symmetry implies that each equilibrium bid is uniformly distributed among the $N - 1$ opponents.

Following the heuristic derivation of (necessary condition for) the symmetric equilibrium bidding function as in Krishna (2009), with no loss of generality consider bidder $i = 1$ who, upon having observed his own value $v_1 = v$, submits bid $b$. Moreover, let $H(x)$ and $h(x) = H'(x)$ be, respectively, the distribution and density function of the random variable $v_{(1)} = \max_{j \neq 1} v_j$. Then, bidder 1’s expected profit is given by

$$E\Pi_1(b; v) = \int_{0}^{\beta^{-1}(v)} (v - b)h(x)dx + \int_{\beta^{-1}(b)}^{\infty} \left(\frac{\beta(x)}{(N-1)} - v\right)h(x)dx \quad (5)$$

Differentiating (5) with respect to $b$, and equalizing it to zero, we obtain

$$\frac{dE\Pi_1(b; v)}{db} = (v - b) \frac{h(\beta^{-1}(b))}{\beta'(\beta^{-1}(b))} - \frac{\beta(\beta^{-1}(b))h(\beta^{-1}(b))}{(N-1)\beta'(\beta^{-1}(b))} + v \frac{h(\beta^{-1}(b))}{\beta'(\beta^{-1}(b))} = 0$$

and since in a symmetric equilibrium it is $v = \beta^{-1}(b)$ it follows that

$$(v - b) \frac{h(v)}{b'} - \frac{bh(v)}{(N-1)b'} + v \frac{h(v)}{b'} = 0$$

from which we immediately obtain
The symmetric equilibrium bidding function (6) exhibits some interesting features, which are also consistent with the “winning losers” spirit of the model with complete information. First it is increasing in \( N \), hence achieves its lowest value at \( N = 2 \), where it is equal to \( b(\nu) = \nu \), that is a fully revealing equilibrium. As the number of co-owners \( N \) gets large bids \( b(\nu) \) tend to \( 2\nu \). That is, as the competition increases, from the perspective of a single player it becomes more likely that some of the co-owners would have a higher value, and so it is optimal to bid above one’s value. The intuition is simple. Consider a bidder with value \( \nu \); then the probability that the maximum value \( v(1) \) is larger than \( \nu \)

\[
P(v(1) > \nu) = 1 - F(\nu)^{N-1}
\]

tends to one as \( N \) gets large. Hence, as the number of co-owners increases it becomes almost certain that one of bidder 1’s competitors will have a value higher than \( \nu \). Therefore for bidder 1 could be profitable to offer a price higher than his own value, to put pressure on his best competitor, though not indefinitely being bounded from above by twice the private value of the object. However, for given \( \nu \) the bid received by each co-owner \( \frac{b(\nu)}{N-1} = \frac{2\nu}{N} \) is decreasing in \( N \) and tends to zero as the number of bidders becomes large. So, if all bidders share the same value \( \nu \) then in equilibrium they would never obtain more than \( \nu \), typically less.

4. Conclusions

In this paper we have characterized the strategic features of an auction mechanism used, for co-ownership resolution of rights over players’ performance, by the Italian Football League. The auction presents a number of interesting and specific features, a main one being that the winner will be the buyer and the loser the seller. For this reason, buyer and seller could only be defined after, and not before, the auction takes place. In a complete information framework we first investigated the original auction mechanism with two players. The equilibrium analysis suggests that “losers can win”, that is not only they never obtain a negative expected payoff but, under certain conditions, their returns could be higher than the winners’. A similar spirit underlies also the incomplete information analysis.

We then extended the framework to any finite number of players and discussed how in such an extension a number of results holding with two players do not carry through. We suggest that this is mostly is due to the fact that with two players the entire winning price goes to the loser, while with more than two players the winning price has to be distributed among the losing bidders. For this reason, a main difference with the two players
framework is that in equilibrium losers, as well as winners, can all obtain negative payoffs since, by unilateral deviation, they can lose even more.

**Acknowledgments.** I would like to thank Luca Pratelli for comments and insights. All errors remain mine.

**References**


[www.legacalcio.it](http://www.legacalcio.it)
